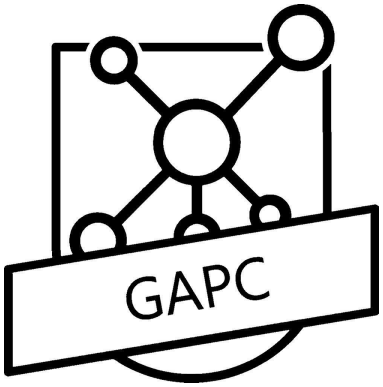
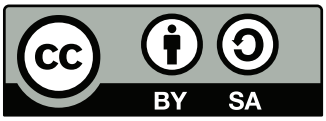
GAPC 2023

Groningen Algorithm Programming Contest 2023



Problems



A A Rod in a Dot

Time limit: 1s

POV: You've just been born to three fathers.

There is no better way to relax than to enjoy a game of darts. However, the poor student Majki does not have a dart board. But Majki decided that this cannot stop him, so he made his own dart board. He was very creative: he took a square plank and attached a bunch of coasters to it. The goal of his game is to hit the coasters with darts he made of small rods.

In order to enjoy his new creation, Majki is now throwing n rods of infinitely small diameter onto a square target of side s. Note that Majki is quite good at throwing rods, so the rods always land straight on the plank. In other words, they never land sideways. The target contains m coasters (circles), the ith of which has radius r_i and center (x_i, y_i). The coasters do not overlap and are fully contained in the square target. Majki always manages to hit the square target and the point of contact between the rod and the square target is uniformly distributed over the square target. Calculate the expected number of times that Majki's rod hits one of the coasters.

Input

The input consists of:

- One line with three integers n, s, and m ($0 \leq n \leq 10^6$, $2 \leq s \leq 10^6$, $0 \leq m \leq 10^6$), the number of rods Majki is throwing, the side length of the square plank, and the number of coasters.
- m lines, each with three integers x_i, y_i, and r_i ($1 \leq x_i, y_i, r_i \leq s - 1$), the center coordinates and the radius of the ith coaster.

Output

Output the expected number of times that Majki's rod hits one of the coaster.

Your answer should have an absolute or relative error of at most 10^{-6}.

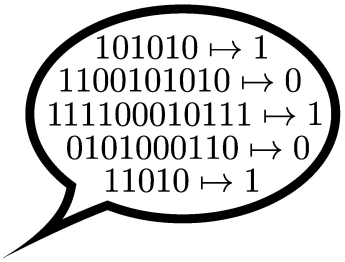
Sample Input 1	Sample Output 1
1 2 1 1 1 1	0.7853981633974483

Sample Input 2	Sample Output 2
5 10 2 1 1 1 5 5 1	0.3141592653589793

Sample Input 3	**Sample Output 3**
3 10 3 2 2 1 6 4 3 2 8 2	1.3194689145077132

B Binary Speakers

Time limit: 2s



The inhabitants of planet Post communicate using functions $f\colon \{0,1\}^k \to \{0,1\}$ ($k \geq 1$) in a similar way as we use words. People learn how to communicate by memorizing a few basic functions and ways of combining them to form more complex ones. Their language is the collection of all binary functions they can express in this way. A person only understands another if she knows all the functions the other knows.

Different regions of this planet teach their inhabitants the same combination mechanisms but possibly different basic functions, so that it is not guaranteed that people from different regions understand each other.

Since the year 1111110011, the global government of planet Post, in an effort to improve communication among regions, established that every region has to teach their inhabitants at least the following functions:

- the *projections* $\pi_k^i\colon \{0,1\}^k \to \{0,1\}$, such that $\pi_k^i(a_1,\ldots,a_k) := a_i$, for all $1 \leq i \leq k$ and all $k \geq 1$; and
- the function $s\colon \{0,1\}^3 \to \{0,1\}$ given by the following table that relates all possible inputs (a,b,c) to the output $s(a,b,c)$:

a	b	c	$s(a,b,c)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

The combination mechanism is *superposition*, meaning that we can form a new function h of n arguments from known functions f (of m arguments) and m functions g_i of n arguments such that for all $a_1,\ldots,a_n \in \{0,1\}$:

$$h(a_1,\ldots,a_n) := f(g_1(a_1,\ldots,a_n),\ldots,g_m(a_1,\ldots,a_n))$$

Mister Bin inhabits a region that only teaches these minimum language features, that is, only the above-defined functions can be combined via superposition.

See some examples of binary functions, named h_1, h_2 and h_3, that Mister Bin understands:

a	b	$h_1(a,b)$	$h_2(a,b)$	$h_3(a,b)$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	0
1	1	1	1	1

Given the set of basic functions beyond the minimum required taught in a region R of planet Post, tell whether Mister Bin fully understands the individuals from R or not. When it is possible, you have to provide expressions in Mister Bin's language corresponding to the basic functions taught in R.

Input

The input consists of:

- One line with an integer N ($1 \leq N \leq 10$), the number of basic functions beyond the minimum taught in region R.
- N groups of lines, each specifying one of these functions as follows:
  - One line with two integers k and z ($1 \leq k \leq 10$, $0 \leq z \leq 2^k$), the number of arguments the function accepts and the number of inputs for which the function outputs 0.
  - z lines, each with k integers $a_1, \ldots, a_k$, denoting a k-tuple $(a_1, \ldots, a_k) \in \{0,1\}^k$ for which the function outputs 0.

Output

For each of the N specified functions $f\colon \{0,1\}^k \to \{0,1\}$, output a line having one of the following forms:

- `no`, if the function is not understood by Mister Bin; otherwise
- `yes,<expr>` , where `<expr>` is f written as a superposition of the basic functions taught to Mister Bin. This string has one of the following forms:
  - `pi`n`(x)`, for $1 \leq n \leq k$ (here `x` represents a k-tuple, since the projections receive k arguments); and
  - `s(`$F1$`,`$F2$`,`$F3$`)`, where $F1$, $F2$ and $F3$ are strings formed in this same way.

The output must have at most 10^7 characters.

Sample Input 1	**Sample Output 1**
1 1 1 0	yes,pi1(x)

Sample Input 2

```
2
2 2
0 1
1 0
2 1
0 0
```

Sample Output 2

```
no
yes,s(pi2(x),pi2(x),pi1(x))
```

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C Cutting Cake

Time limit: 1s

Credit: Dave Morris
CC-BY-NC-SA-2.0.

Charles is giving a birthday party for k people (including himself). He has a rectangular cake that needs to be cut into k rectangular pieces with the same height and width. Due to the pattern on top of the cake, rotating the pieces is not allowed. Since it's Charles' birthday, he's not going to do anything himself, so he asks Vincent to cut the cake using only cuts that are parallel to the sides of the cake.

When Vincent hears the task, he complains: "Why did you have to give a birthday party with exactly k people? Now I need to make a lot of cuts!". Charles thinks about this and agrees: "You are right. It is fine to cut the cake into a few additional pieces (which still all need to be equal), as long as they are at most s% smaller than when cutting the cake into exactly k pieces."

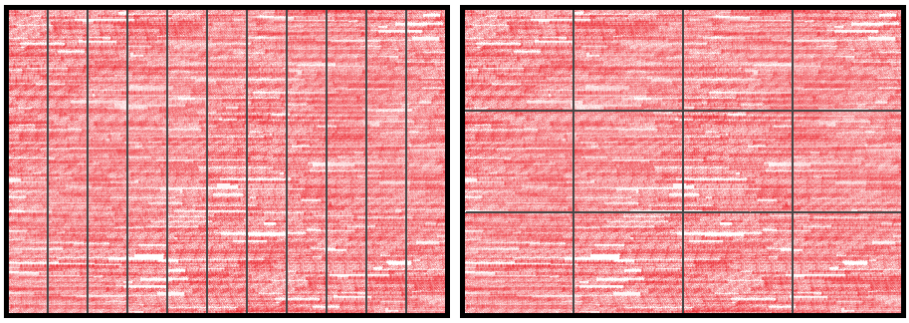


Figure C.1: Illustration of Sample Input/Output 1 and 2. In Sample Input 1, 11 pieces are needed and they are not allowed to be smaller, so Vincent needs to make 10 cuts. In Sample Input 2, the pieces are allowed to be at most 10% smaller, so Vincent can make 12 pieces using 5 cuts.

Input

The input consists of:

- One line with two integers k and s ($1 \leq k \leq 10^9$, $0 \leq s \leq 50$) the number of people and the maximum percentage that the pieces are allowed to be smaller.

Output

The minimum number of cuts that needs to be made to cut the cake in $c \geq k$ equal pieces, using only cuts that are parallel to the sides of the cake, and where each piece is at most s% smaller than when cutting the cake in exactly k equal pieces.

Sample Input 1	Sample Output 1
11 0	10

Sample Input 2	**Sample Output 2**
11 10	5

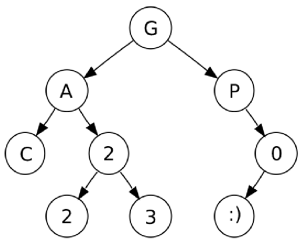
Sample Input 3	**Sample Output 3**
101 42	19

D Discrete Structures

Time limit: 1s

The course Discrete Structures is a fundamental course in the first block of the first year of the Gaming, AI, and Practical Coding degree. In order to pass, students have to:

- Attend the lectures. If you don't, you might lose on very important information.
- Make three homeworks. The best two of them count equally towards 30% of the final grade.
- Make the midterm, which counts towards 20% of the final grade.
- Make the final exam, which counts towards 50% of the final grade.



This is not a binary search tree.

In order to help your fellow students, you decided to write a program that will compute the final grade for the course, given their grades.

Input

The input consists of five lines, each with one floating point number x ($1 \leq x \leq 10$), with at most 10 digits after the decimal point:

- One line containing the grade for the first homework.
- One line containing the grade for the second homework.
- One line containing the grade for the third homework.
- One line containing the grade for the midterm.
- One line containing the grade for the final exam.

Output

Output the final grade for the course.

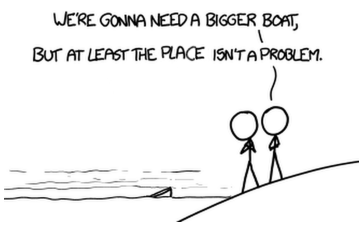
Your answer should have an absolute or relative error of at most 10^{-6}.

Sample Input 1	Sample Output 1
8.5 4.1 8.9 9.2 10	9.45

Sample Input 2	**Sample Output 2**
8.4 9.0 7.2 6.8 5.9	6.92

E Epic Party on a Boat

Time limit: 1s



Source: what-if.xkcd.com.

Wojti, Majki, and Franki go to Delft for Northwestern Europe Regional Contest. Since it's winter, they decide to sleep on a boat without any heating or appropriate sleeping bags. As soon as they realise their mistake, they decide to invite friends over in order to make it a little warmer. Unfortunately, the boat is very small and can fit a limited number of people.

They asked their Airbnb host how many people can fit, but he turned out to be very greedy and told them: "The number of additional people that the boat can sustain is between 1 and n. You can try to guess the number and I will tell you if the true one is smaller, equal to or larger than your guess. However, each time you guess number k, I will charge you k Grumpy Angry Panda Coins. Moreover, you always have to guess until I answer 'equal', even if you can already deduce the answer from the earlier guesses.". Wojti, Majki, and Franki are wondering if that deal is worth it. They want to calculate the worst-case cost (in Grumpy Angry Panda Coins) they would have to pay if they were guessing numbers optimally.

Input

The input consists of:

- One line with an integer n ($1 \leq n \leq 100$), the largest possible number of people that the boat can sustain in addition to Wojti, Majki, and Franki.

Output

Output the best worst-case cost of guessing the number of people that the boat can sustain.

Sample Input 1	Sample Output 1
3	5

Sample Input 2	Sample Output 2
4	7

Sample Input 3	Sample Output 3
5	9

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F Flatland Zoo

Time limit: 1s

Welcome to flatland, where everything is flat, except for Jimmy, a sphere of insignificant dimensions. There is a zoo in flatland full of beautiful flat animals. The zoo consists of a rectangular grid composed of $m \times n$ square spots of same size. Each spot mimics beautifully a natural flat environment for the flat animal living there. In order to visit a spot, one only needs to step into its interior.

Time is running fast, and Jimmy needs to visit as many spots as he can. After thinking for a while, he decided to do the following: start at the entrance in the bottom-left corner of the zoo and walk straight until reaching the exit in the top-right corner.

Can you tell how many spots Jimmy will be able to visit this way?

Input

The input consists of:

- One line containing two integers m and n ($1 \leq m, n \leq 10^{15}$), the dimensions of the zoo.

Output

Output the number of spots Jimmy will be able to visit.

Sample Input 1	Sample Output 1
3 3	3

Sample Input 2	Sample Output 2
2 5	6

Sample Input 3	Sample Output 3
20 10	20

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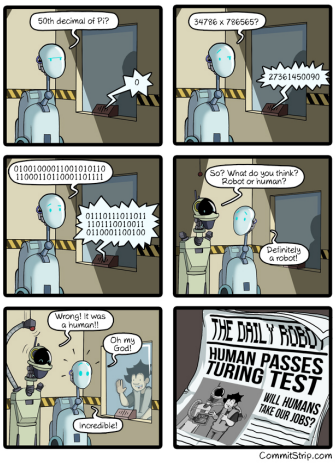
G Gruesome CAPTCHAs

Time limit: 2s



Source: www.commitstrip.com

As the friction between humans and AI has been lately increasing, you have found that your favourite website with AI generated content introduced new CAPTCHAs. Although you were prepared for selecting all the fire hydrants or even deciphering an obscure text, anti-human CAPTCHA is definitely not what you expected.

You are presented with a directed graph having no self-loops, no double edges, and no bidirectional edges. After clicking 'continue', the edges disappear and the nodes of the graph start flashing, one at a time. When a node flashes, all the outgoing and ingoing connections change their direction. After a series of flashes you are asked to write which node (if any) is the centre of the graph. A node that is a centre has a direct outgoing connection to every other node of the graph. Before you have time to write anything, the nodes start flashing again, further altering the connections.

Although you are determined to break the CAPTCHA, the graphs presented are huge and you only have a second to succesfully complete it. Thankfully, as in every website for robots, you can connect to the website through an API, so you may automate the process of solving the CAPTCHA.

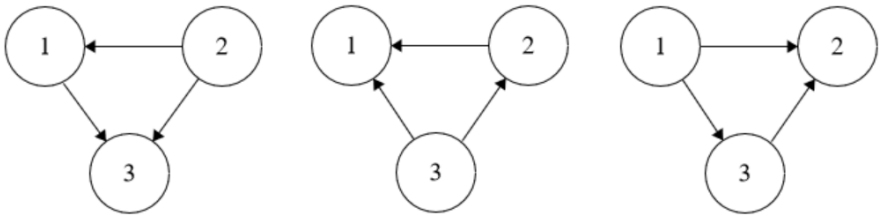


Figure G.1: Illustration of Sample Input/Output 1. The input graph is the graph on the left. After the first series of flashes, all edges connected to node 3 change direction. Now 3 is a graph centre. After the second series of flashes, all edges connected to node 1 change direction. Now 1 is a graph centre.

Input

The input consists of:

- One line with three integers n, e, q ($3 \leq n \leq 10^3$, $3 \leq e \leq 3 \cdot 10^5$, $1 \leq q \leq 10^4$), the number of nodes, the number of edges between them, and the number of queries. Each node is assigned a distinct index from 1 to n.
- e lines, each containing two distinct integers a, b ($1 \leq a, b \leq n$), indicating an edge from node a to node b.

- q groups of 2 lines, each specifying a query as follows:
  - One line with an integer t ($1 \leq t \leq 10^4$), indicating the number of changes in the query.
  - One line with t integers x ($1 \leq x \leq n$), the nodes whose edges change direction.

It is guaranteed that in total over all q queries, there are at most 10^4 nodes whose edges change direction.

Output

For each of the q queries, print `impossible` if the graph has no centre, or the index of the node that is the centre of the graph.

Sample Input 1

```
3 3 2
2 1
1 3
2 3
1
3
1
1
```

Sample Output 1

```
3
1
```

Sample Input 2

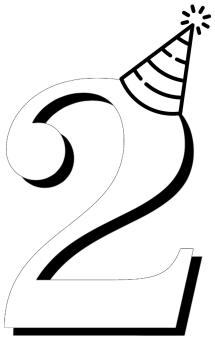
```
4 4 2
4 2
3 2
1 4
2 1
1
2
3
3 2 4
```

Sample Output 2

```
impossible
2
```

H Hasty Guesses

Time limit: 1s



For their 22nd birthday, your friend invited you to their party whose theme is the number two. Upon arriving to the party, you find that everyone is playing a strange game. Your friend is employed at Researchable, so he really likes numbers and therefore the game is about guessing numbers. One person says a target number, and then everyone writes down a number on a piece of paper. All the paper sheets are collected and the winner(s) of the round are determined. The second closest number to the target wins. As multiple people can choose the same number, there may be more than one winner.

The game seems to be super popular but it takes too much time to determine the winning number and people start second-guessing the choice. Can you help automating the process of determining the winning number?

Input

The input consists of:

- One line with two integers n, k ($2 \leq n \leq 10^5$, $0 \leq k \leq 10^5$), the number of people playing the game, and the target number.
- One line with n integers x ($0 \leq x \leq 10^5$), representing the guesses.

It is guaranteed that there will be at least two distinct guesses.

Output

Output the second closest number to the target from the set of numbers k.

If two numbers are equally close to the target, treat the smaller as being closer.

Sample Input 1	Sample Output 1
3 10 7 15 12	7

Sample Input 2	Sample Output 2
5 20 17 17 15 23 50	23

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I International Interpolation

Time limit: 1s

It is your first day at work in your new job, what an exciting day! You have recently got a job at the Student Support Desk of RUG, sometimes called the Glorious Adolescent Problem-solving Center (GAPC).

Your first task is to print letters of admission for new international students that are coming to the RUG. Unfortunately, this can be quite problematic because international students have many non-standard characters in their names that your computer cannot handle.

However, as you also work part-time at Klippa, you have a bunch of ideas on how to work with documents, so you quickly found a solution! What if instead of displaying these characters you find the letter that is in the alphabet exactly halfway between the two surrounding letters and replace it with that? And if there are two candidates, you can just pick the one that goes first in the alphabet. If the character appears at the start of the word in a name, then you can just put "`a`" and if it is at the end of the word, you can insert "`z`". I am sure nobody will notice, and internationals are used to having their names butchered anyway...

However, if there are two non-standard characters somewhere in a row, then you should not print the name, as you do not have enough information to infer from. (This rule also applies to adding letters at the start and end of the word.)

Input

The input consists of:

- One line with an integer n ($1 \leq n \leq 200$), the number of names in the input sequence.
- n lines each containing a non-empty string of maximum length 10^3, representing a name of an international student with some characters replaced by "`#`" to indicate the presence of a non-standard character.

Output

Output the modified names, each on a separate line.

If there are two non-standard characters in a row output "`impossible`" instead.

Sample Input 1	Sample Output 1
2 a#d a##e	abd impossible

Sample Input 2	**Sample Output 2**
2 a#c #a#e#	abc aacez

J Just in Time

Time limit: 2s

Source: www.asml.com

Arnold is a production manager at a factory at ASML. Recently, there has been increasing demand for lithographic machines, so Arnold receives many orders every day. Thankfully, he receives the orders ahead of time, specifying when each order should arrive. Also, Arnold has a large group of workers he can rely on, since handling n orders would not be easy.

Since the machines are quite fragile, they cannot sit in the factory after they have been finished, they must be finished exactly at the time they need to be delivered – or as Arnold likes to say – just in time. The workers cannot interrupt working on a machine (it is too complicated), and for each machine, there can only be one worker working on it. Each order takes some time to be prepared, and they also take some time to be delivered. The orders of machines are not necessarily listed in order (Arnold receives them as clients order from the website). Arnold has to figure out how many people he needs to call in to finish all the orders in time. Can you help him calculate that?

Input

The input consists of:

- One line with an integer n ($0 \leq n \leq 10^5$), the number of orders Arnold has received.
- n lines, the ith of which contains three integers a_i, b_i and c_i ($a_i \geq b_i + c_i$, $2 \leq a_i \leq 10^9$, $b_i, c_i \geq 1$), the time when order i must be delivered, how long it takes to process the order, and how long it takes to deliver the order.

Output

Output the number of people needed to finish the orders just in time.

Sample Input 1	Sample Output 1
2 10 5 5 9 1 8	2

Sample Input 2	Sample Output 2
3 9 3 2 10 2 2 3 2 1	2